# Effective heat conductivity of fuel element bundles and steam generator tube bundles 

V. Fedotovsky *, A. Orlov<br>State Scientific Center of the Russian Federation, Institute for Physics and Power Engineering, Obninsk, Russia

## A R T I C L E I N F O

## 1. Effective heat conductivity

Effective heat conductivity of rod and tube bundles is one of thermophysical properties necessary for calculation of thermo hydraulic characteristics of fuel elements, heat exchangers and steam generators.

The term effective heat conductivity considers not only heat conductivity of coolant and rod and tube materials respecting their geometrical position in bundles but also effects of turbulent heat conductivity, interchannel mixing, phase change, connected with processes of freezing and melting of liquid metal coolants and so on.

Numerous literatures are dedicated to research of effective heat conductivity or in more common sense generalized conductivity [2]. Wide number of theoretical and experimental results for heterogeneous materials of various structure are available [1,6], many of which are not systematized or generalized.

In SSC of RF IPPE formula for effective heat conductivity of a medium with inclusions of arbitrary shape was received on the basis of conception of effective vibrodynamic properties of heterogeneous mediums [4] and thermomechanical analogy [5]
$\frac{\lambda}{\lambda^{*}}=1+\frac{(1+\gamma)\left(\lambda / \lambda_{0}-1\right) \varphi}{\lambda / \lambda_{0}+\gamma}$,
here $\gamma$ is the form-factor of inclusions that in sense of hydrodynamics means added mass, $\varphi$ is the volumetric concentration of inclusions, $\lambda$ is the medium heat conductivity coefficient, $\lambda_{0}$ is the inclusions heat conductivity coefficient.

[^0]In general case formula (1) describes dependence of effective heat conductivity tensor of anisotropic heterogeneous materials where form factor $\gamma$ is a 2 nd rank tensor.

Particularly for not highly concentrated heterogeneous mediums with parallel and uniformly distributed cylindrical inclusions the formula was received from the formula (1) for transversal component of heat conductivity tensor
$\frac{\lambda^{*}}{\lambda}=\frac{(1-\varphi)+\left(\lambda / \lambda_{0}\right)(1+\varphi)}{(1+\varphi)+\left(\lambda_{0} / \lambda\right)(1-\varphi)}$
known as Maxwell formula.
For closely packed systems of cylinders from formula (1) the following formulas for effective heat conductivity of systems forming rectilinear triangular or square grids were received. In particular for systems with nonheatconductive inclusions these formulas would be [3]
$\frac{\lambda^{*}}{\lambda}=\frac{X \sqrt{X-1}}{\sqrt{6}}\left[\operatorname{arctg}\left(\sqrt{\frac{2}{X-1}} \operatorname{tg} \frac{\pi}{12}\right)\right]^{-1}$,
$\frac{\lambda^{*}}{\lambda}=\frac{X \sqrt{X-1}}{\sqrt{2}}\left[\operatorname{arctg}\left(\sqrt{\frac{2}{X-1}} \operatorname{tg} \frac{\pi}{8}\right)\right]^{-1}$,
here $X$ is relative grid spacing.
Numerical calculations were performed to determine the area of application of general formula (1) and a number of its particular cases.

This report introduces results of mathematical modeling of effective heat conductivity of transversally anisotropic rod bundles in solid conductive medium formed by cylindrical rods in rectangular and triangular grids compressed or stretched in heat flow direction. Similar numerical modeling was performed anisotropic bundles of elliptic cylinders and also for disperse-armed


Fig. 1. Comparison of received results with analytical solutions and numerical results received before.


Fig. 2. Comparison of received results with analytical solutions and numerical results received before.
materials with spherical and ellipsoidal inclusions. On the basis of quite precise numerical solutions of stationary heat conductivity problems in representative cells by finite elements method dependencies for components of effective heat conductivity tensor in principal axes were received and comparison with formulas received analytically for heterogeneous mediums of various structure (Figs. 1 and 2) was performed. As a result of comparison limits of application of various analytic formulas and methods of their specification were determined.

## 2. Analysis of results

Infinite bundle of parallel cylindrical rods fitted in square grid is situated in solid conductive medium.

Lets assign: $\lambda^{*}$ is the effective heat conductivity of medium with rods in direction transversal to rod axes, $\lambda_{1}$ is the heat conductivity of medium, $\lambda_{2}$ is the heat conductivity of rod material. In figures shown below you can compare results received during this work with formulas and results of numerical calculations received earlier by Shmid.

Calculated effective conductivity parameter for rods in square and triangular packages for various combinations of conductivity parameters are given in Tables 1-6.

In Fig. 1 dependencies of effective heat conductivity in case of big difference in heat conductivity of inclusions and medium are shown. One can see that with the increase in inclusions concentration our calculations significantly specify Maxwell and other formulas.

From these figures one can see that at low and medium concentrations of inclusions effective heat conductivity is fairly accurately described by the formula (4), Shmid's numerical calculations and our numerical calculations. However with the increase in inclusions concentration formulas (2) and (3) have to be modified with more precise coefficients of added mass.

Due to obvious reasons Shmid did not perform numerical calculations in the area of high concentrations (see Fig. 3).

Results of calculations given in this work considerably supply previous results received numerically, analytically and this results help to better understand the nature of processes especially in an area of high concentrations.

Table 1
Effective conductivity parameter for rods in square package

| $\varphi$ | 0.0491 | 0.1963 | 0.3846 | 0.5027 | 0.6363 | 0.6647 | 0.6938 | 0.7234 | 0.7534 | 0.7698 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1.4715 | 2.1997 | 2.9727 | 4.6278 | 5.265 | 6.1535 | 7.523 | 10.088 | 12.28 | 18.584 |

Table 2
Effective conductivity parameter for rods in triangular package

| $\varphi$ | 0.0567 | 0.2267 | 0.4444 | 0.5804 | 0.7346 | 0.7676 | 0.8013 | 0.8356 | 0.871 | 0.8889 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\lambda_{2} / \lambda_{1}=50$ | 1.1153 | 1.552 | 2.4738 | 3.5139 | 5.9125 | 6.9345 | 8.1273 | 10.173 | 13.906 | 18.408 |

Table 3
Effective conductivity parameter for rods in square package

| $\varphi$ | 0.0491 | 0.1963 | 0.3846 | 0.5027 | 0.6363 | 0.6647 | 0.6938 | 0.7234 | 0.7534 | 0.7698 | 0.7854 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\lambda_{2} / \lambda_{1}=5$ | 1.0682 | 1.3052 | 1.7012 | 2.0395 | 2.5487 | 2.6908 | 2.853 | 3.0412 | 3.2595 | 3.3777 | 3.5219 |

Table 4
Effective conductivity parameter for rods in triangular package

| $\varphi$ | 0.0567 | 0.2267 | 0.4444 | 0.5804 | 0.7346 | 0.7676 | 0.8013 | 0.8356 | 0.871 | 0.8889 | 0.9069 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\lambda_{2} / \lambda_{1}=5$ | 1.0787 | 1.3533 | 1.834 | 2.2581 | 2.9244 | 3.1187 | 3.3057 | 3.5566 | 3.8523 | 4.0462 | 4.1877 |

Table 5
Effective conductivity parameter for rods in square package

| $\varphi$ | 0.0491 | 0.1963 | 0.3846 | 0.5027 | 0.6363 | 0.6647 | 0.6938 | 0.7234 | 0.7534 | 0.7698 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.7854 |  |  |  |  |  |  |  |  |  |  |
| $\lambda_{2} / \lambda_{1}=1 / 3$ | 0.952 | 0.8191 | 0.6735 | 0.5912 | 0.5088 | 0.4912 | 0.4735 | 0.4556 | 0.4385 | 0.4299 |

Table 6
Effective conductivity parameter for rods in triangular package

| $\varphi$ | 0.0567 | 0.2267 | 0.4444 | 0.5804 | 0.7346 | 0.7676 | 0.8013 | 0.8356 | 0.871 | 0.8889 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\lambda_{2} / \lambda_{1}=1 / 3$ | 0.94467 | 0.79732 | 0.63845 | 0.55024 | 0.46286 | 0.44534 | 0.42933 | 0.40976 | 0.39021 | 0.38003 |



Fig. 3. Comparison of received results with analytical solutions and numerical results received before.

## 3. Conclusion

Our numerical calculations have given us new, more accurate data especially in cases of high concentrations (up to contact) of rod arrangements. High accuracy allows to consider the results received as standard for comparison with results of approximate mathematical modeling and conclusions relative to accuracy of models and domain of applicability.

From comparison of received numerical solutions to effective heat conductivity of heterogeneous systems in the form of transversally anisotropic rod bundles with results of calculations by means of approximate mathematical models following conclusions can be made:

1. Results of these calculations very well agree with the results of analytical calculations.
2. In an range of high concentrations these calculations significantly supplement the specified formulas (3) for effective heat conductivity.
3. This way of calculations allows to receive more precise results for effective heat conductivity of more complex systems that do not have precise analytical solutions which is shown on the example of square arrangement of ellipsoids.

## References

[1] A.F. Chudnovsky, Thermal Characteristics of Disperse Materials, Pfizmatgiz, 1962.
[2] G.N. Dulnev, Yu.P. Zarichnyak, Heat conductivity of mixtures and composite materials, Reference Book, Leningrad: Energy. 1974.
[3] V.S. Fedotovsky, Dynamics of heterogeneous media and hydroelastic rod systems under vibrations, Dissertation, IPPE, Obninsk, 1991.
[4] V.S. Fedotovsky, in: Proceeding of the First Russian National Conference on Heat Exchange, Part 2, M., MEI, vol. 10, 1994, p. 116.
[5] V.S. Fedotovsky, in: Proceeding of the Second Russian National Conference on Heat Exchange, M., MEI, vol. 7, 1998, p. 232.
[6] Shmid I., Calculation of effective transversal heat conductivity coefficient for regular triangular and square grids. Report, Nuclear Research Institute, Rjej, 1984.


[^0]:    * Corresponding author.

    E-mail address: fedotovsky@ippe.obninsk.ru (V. Fedotovsky).

